

Evolution of primordial perturbations and a fluctuating decay rate.

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Abstract

We present a gauge invariant formalism to study the evolution of curvature perturbations during decay of the inflaton or some other field that dominates the energy density. We specialize to the case where the total curvature perturbation arises predominantly from a spatially fluctuating decay rate. Gaussian fluctuations in the decay rate are present if the inflaton coupling or mass depends on some light field, which fluctuates freely during inflation. The isocurvature fluctuations of the light field seeds the total curvature perturbations, whose amplitude freezes after inflaton decay. We also analyze the resulting curvature perturbation starting with non-negligible values of the initial inflaton curvature. Lastly, we study the effects of the energy density stored in the light field.

I. INTRODUCTION

Inflation is the main contender for an explanation of the observed adiabatic density perturbations with a nearly scale invariant spectrum [1]. However, recently, alternative mechanisms for generating the density perturbations have also been discussed. In the curvaton scenario, isocurvature perturbations of some light “curvaton” field are converted into adiabatic perturbations in the post-inflationary universe [2].

Another interesting proposal is that the perturbations could be generated from the fluctuations of the inflaton coupling to the Standard Model degrees of freedom [3, 4, 5]. It has been argued that the coupling strength of the inflaton to ordinary matter or the inflaton mass, instead of being a constant, could depend on the vacuum expectation values (VEV) of the various fields in the theory. If these fields are light during inflation their quantum fluctuations will lead to spatial fluctuations in the inflaton decay rate. As a consequence, when the inflaton decays, adiabatic density perturbations will be created because fluctuations in the decay rate translate into fluctuations in the reheating temperature. A variation of the above scenario is that the inflaton decays into heavy particle, whose decay is governed by a varying decay rate. If these particles come (close) to dominating the energy density of the universe, curvature perturbations are produced [6]. The light fields whose fluctuations leads to a fluctuating decay rate could be normal matter fields, such as the flat directions in the minimal supersymmetric Standard Model (MSSM), or a right handed neutrino [7].

In this paper we will discuss the evolution of the curvature perturbation during decay, in the case of a fluctuating decay rate. We will use the gauge-invariant formalism for metric perturbations as developed by Bardeen [8], for a review see Ref. [9]. The curvature perturbation on large scales (defined on a uniform density spatial hypersurface) ζ remains constant for purely adiabatic perturbations. But ζ can change on large scales due to a non-adiabatic pressure perturbation. In a multi-fluid system such pressure perturbations can be important in the presence of relative entropy perturbations. Said in another way, entropy (or isocurvature) perturbations can feed the curvature perturbation. Therefore, to compute the final perturbation spectrum in a multi fluid system it is necessary to keep track of the evolution and perturbations of various fluids.

In the case of a fluctuating decay rate, the multi fluid system consist of the decaying field (the inflaton, or some other field which has dominant energy density), the radiation bath,

and the light or flat direction field that is responsible for the fluctuating decay rate. The relative curvature perturbation between the inflaton and the flat direction acts as a source for the total curvature perturbations. This can be understood intuitively from the fact that fluctuations in the inflaton decay rate leads to fluctuations in the reheat temperature of the universe, given by $T_{\text{rh}} \sim \lambda \sqrt{m_\phi M_p}$, where m_ϕ is the mass of the inflaton and $M_p = 2.436 \times 10^{18}$ GeV is the reduced Planck mass. The fluctuations in Γ can be translated into fluctuations in the energy density of a thermal bath with $\delta\rho_\gamma/\rho_\gamma = -(2/3)\delta\Gamma/\Gamma$ [3]. The factor 2/3 appears due to red-shift of the modes during the decay of the inflaton whose energy still dominates.

The inflaton decay rate is $\Gamma \sim m_\phi \lambda^2$. The decay rate fluctuates if either λ or m_ϕ is a function of a fluctuating light field. The former case can e.g. arise in the MSSM through terms in the superpotential of the form

$$W \ni \lambda_h \phi \bar{H} H + \phi \frac{q}{M} q q + \phi \frac{q_c}{M} q_c q_c + \phi \frac{h}{M} q q_c, \quad (1.1)$$

with the inflaton field ϕ a standard model singlet, H and \bar{H} the two Higgs doublets, and q and q_c the quark and lepton superfields and their anti-particles. M is some cut-off scale which could be the GUT scale or the Planck scale. In this paper we will not be concerned with the particulars of the decay products and the light fields. What we will take from the above example though, is that there can be two kinds of couplings:

$$\lambda = \begin{cases} \lambda_0 \left(1 + \frac{S}{M} + \dots\right), & \text{direct decay.} \\ \lambda_0 \left(\frac{S}{M} + \dots\right), & \text{indirect decay,} \end{cases} \quad (1.2)$$

with the ellipses standing for higher order terms. In the MSSM example, the “direct” decay mode corresponds to inflaton decay into Higgs fields, whereas the “indirect” decay mode corresponds to decay into (s)quarks and anti-(s)quarks. S is the expectation value of a light fluctuating field. Effective couplings of this form can result from integrating out heavy particles. Another possibility is that the inflaton, or some other field that dominates the energy density of the universe (e.g. non-relativistic inflaton decay products), has an effective mass generated through a coupling to the flat direction: $m_\psi = \lambda S$. The fluctuation in the decay rate for the various cases is

$$\frac{\delta\Gamma}{\Gamma} = \begin{cases} 2\frac{\delta S}{M}, & \text{direct decay.} \\ 2\frac{\delta S}{S}, & \text{indirect decay.} \\ \frac{\delta S}{S}, & \text{fluctuating mass.} \end{cases} \quad (1.3)$$

II. BACKGROUND EQUATIONS

The background equations of motion governing the dynamics of the inflaton, the radiation bath, and the flat direction field in a Robertson Walker universe are

$$\dot{\rho}_\phi = -\rho_\phi(3H + \Gamma), \quad (2.1)$$

$$\dot{\rho}_\gamma = -4H\rho_\gamma + \Gamma\rho_\phi, \quad (2.2)$$

$$\dot{\rho}_S = -3H\rho_S(1 + \omega). \quad (2.3)$$

where a dot denotes differentiation w.r.t. coordinate time. The Hubble parameter $H = \dot{a}/a$, with a the scale factor of the universe, is given by

$$H^2 = \frac{1}{3M_P^2}(\rho_\phi + \rho_\gamma + \rho_S). \quad (2.4)$$

Here $\omega = p_S/\rho_S$ is the equation of state parameter for the flat direction field S . When $H \gg m_S$, the flat direction field is over damped and remains effectively frozen, and $\omega \rightarrow -1$. For $H \lesssim m_S$, the flat direction field oscillates in its potential. For a quadratic potential $\omega = 0$ averaged over one oscillation. To determine ω and the decay width $\Gamma(S)$ we need the evolution of S :

$$\ddot{S} + 3H\dot{S} + V_{,S} = 0 \quad (2.5)$$

with $V_{,S} = \partial V/\partial S$. Then

$$\omega = \frac{p_S}{\rho_S} = \frac{\dot{S}^2/2 - V(S)}{\dot{S}^2/2 + V(S)}. \quad (2.6)$$

It is useful to introduce the following dimensionless parameters

$$\Omega_\alpha = \frac{\rho_\alpha}{\rho}, \quad s = \frac{S}{m}, \quad g = \frac{\Gamma}{m_\phi}, \quad h = \frac{H}{m_\phi}. \quad (2.7)$$

We leave the mass scale m unspecified for now, it can be taken m_S . Note however that if finite temperature effects or effects of non-renormalizable operators are taken into account, then m_S is time-dependent and we cannot set $m = (m_S)_{\text{eff}}$. Further, we eliminate Ω_γ from all equations by use of the Friedman constraint:

$$\Omega_S + \Omega_\phi + \Omega_\gamma = 1 \quad (2.8)$$

The dimensionless background equations are then

$$\Omega'_\phi = \Omega_\phi \left(1 - \frac{g}{h} - \Omega_\phi + \Omega_S(3\omega - 1) \right) \quad (2.9)$$

$$\Omega'_S = \Omega_S \left((1 - 3\omega) - \Omega_\phi + (3\omega - 1)\Omega_S \right) \quad (2.10)$$

$$h' = \frac{h}{2} \left(-4 + \Omega_\phi + (1 - 3\omega)\Omega_S \right). \quad (2.11)$$

and

$$s'' + \left(\frac{h'}{h} + 3 \right) s' + \left(\frac{V_{,s}(s)}{h^2} \right) s = 0. \quad (2.12)$$

Here, prime denotes differentiation with respect to the number of e-foldings $N \equiv \ln a$.

III. GAUGE INVARIANT PERTURBATIONS

We are interested in the long wavelength regime of the perturbations where the comoving scale is much larger than the Hubble horizon. Following Ref. [8] we define the curvature perturbation ζ on spatial slices of uniform density ρ with the line element ¹

$$ds^2 = a^2(t) (1 + 2\zeta) \delta_{ij} dx^i dx^j, \quad (3.1)$$

where a is the scale factor. The time evolution of the curvature perturbation on large scales is [10, 11, 12]

$$\dot{\zeta} = -\frac{H}{\rho + P} \delta P_{\text{nad}}, \quad (3.2)$$

where $P_{\text{nad}} \equiv \delta P - c_s^2 \delta \rho$ is the non-adiabatic pressure perturbation. The adiabatic sound speed is $c_s^2 = \dot{P}/\dot{\rho}$, where P and ρ are the total pressure and energy density respectively. For a single field $\delta P_{\text{nad}} = 0$, and therefore on large scales the curvature perturbation is pure adiabatic in nature with $\zeta = \text{constant}$.

The total curvature perturbation Eq. (3.2) can be written in terms of the various components

$$\zeta = \sum_{\alpha} \frac{\dot{\rho}_{\alpha}}{\dot{\rho}} \zeta_{\alpha}. \quad (3.3)$$

In our case $\alpha = \phi, S, \gamma$. Isocurvature or entropy perturbations describe the difference between the curvature perturbations [11]

$$S_{\alpha\beta} = 3(\zeta_{\alpha} - \zeta_{\beta}) = -3H \left(\frac{\delta\rho_{\alpha}}{\dot{\rho}_{\alpha}} - \frac{\delta\rho_{\beta}}{\dot{\rho}_{\beta}} \right). \quad (3.4)$$

With the help of Eqs. (3.3,3.4), we can obtain a useful relationship

$$\zeta_{\alpha} = \zeta + \frac{1}{3} \sum_{\beta} \frac{\dot{\rho}_{\beta}}{\dot{\rho}} S_{\alpha\beta}, \quad (3.5)$$

¹ Linear perturbations around a flat FRW metric can be defined by the line element $ds^2 = -(1 + 2\phi)dt^2 + 2aB_{,i}dtdx^i + a^2 [(1 - 2\psi)\delta_{ij} + 2E_{,ij}] dx^i dx^j$, where we have used the notation of Ref. [9] for the gauge dependent curvature perturbation ψ , the lapse function ϕ , and the scalar shear $\chi = a^2 \dot{E} - aB$. The quantity ζ is related to the curvature perturbation ψ , on a generic slicing, by $\zeta = -\psi - H\delta\rho/\dot{\rho}$.

where $\alpha, \beta = \phi, S, \gamma$.

In the presence of more than one field the non-adiabatic pressure perturbation can be non-zero $\delta P_{\text{nad}} \neq 0$, and therefore the total curvature perturbations evolve in time. The non-adiabatic pressure has two contributions, one is from the intrinsic pressure perturbations and the other from the relative pressure perturbations [11, 12].

$$\delta P_{\text{nad}} \equiv \sum_{\alpha} \delta P_{\text{intr},\alpha} + \delta P_{\text{rel}}, \quad (3.6)$$

where $\delta P_{\text{intr},\alpha} \equiv \delta P_{\alpha} - c_{\alpha}^2 \delta \rho_{\alpha}$, and $c_{\alpha}^2 \equiv \dot{P}_{\alpha} / \dot{\rho}_{\alpha}$ is the adiabatic speed of sound for a respective fluid. Note that the intrinsic pressure perturbation vanishes for a fixed equation of state. Thus, $\delta P_{\text{intr},\gamma} = 0$, and (during the stage of inflaton oscillations) $\delta P_{\text{intr},\phi} = 0$. For a fixed ω parameter, also the intrinsic pressure perturbation for the flat direction vanishes $\delta P_{\text{intr},S} = 0$. However the relative pressure perturbation is non-zero and can be expressed in terms of the various entropy perturbations as [11]

$$\delta P_{\text{rel}} \equiv -\frac{1}{6H\dot{\rho}} \sum_{\alpha,\beta} \dot{\rho}_{\alpha} \dot{\rho}_{\beta} (c_{\alpha}^2 - c_{\beta}^2) S_{\alpha\beta}. \quad (3.7)$$

Using Eqs. (3.2,3.3,3.4,3.7), the evolution of the total curvature perturbation becomes

$$\dot{\zeta} = \frac{H}{\dot{\rho}^2} \left(\frac{1}{3} \dot{\rho}_{\phi} \dot{\rho}_{\gamma} S_{\phi\gamma} - (\omega - \frac{1}{3}) \dot{\rho}_S \dot{\rho}_{\gamma} S_{S\gamma} + \omega \dot{\rho}_{\phi} \dot{\rho}_S S_{\phi S} \right). \quad (3.8)$$

During the decay of the inflaton in relativistic species there is non-adiabatic energy transfer from the inflaton to the radiation bath, see Eqs. (2.1,2.2):

$$Q_{\phi} = -\Gamma \rho_{\phi}, \quad (3.9)$$

$$Q_{\gamma} = \Gamma \rho_{\phi}. \quad (3.10)$$

Due to fluctuations in Γ , the non-adiabatic energy transfer is also subject to perturbations. Therefore, the evolution of the individual curvature perturbations ζ_{ϕ} and ζ_{γ} is determined not only by non-adiabatic pressure perturbations but also by the non-adiabatic energy transfer perturbations $\delta Q_{\text{nad},\alpha}$. Analogous to the non-adiabatic pressure perturbations, these can be split in an intrinsic and relative part [11, 12]

$$\delta Q_{\text{nad},\alpha} \equiv \delta Q_{\text{intr},\alpha} + \delta Q_{\text{rel},\alpha}. \quad (3.11)$$

The intrinsic non-adiabatic energy transfer is defined as

$$\delta Q_{\text{intr},\alpha} \equiv \delta Q_{\alpha} - \frac{\dot{Q}_{\alpha}}{\dot{\rho}_{\alpha}} \delta \rho_{\alpha}, \quad (3.12)$$

whose contribution is zero if Q_α is a function of local energy transfer $\delta Q_\alpha = \dot{Q}_\alpha \delta \rho_\alpha / \dot{\rho}_\alpha$ [11, 12]. The non-zero contributions are

$$\delta Q_{\text{int},\phi} = -\frac{\dot{\Gamma}}{3H} \rho_\phi S_{\phi S}, \quad (3.13)$$

$$\delta Q_{\text{int},\gamma} = \frac{\dot{\Gamma}}{3H} \rho_\phi S_{\gamma S} - \frac{\Gamma}{3H} \dot{\rho}_\phi S_{\phi\gamma}, \quad (3.14)$$

where we have used $\delta\Gamma/\dot{\Gamma} = \delta S/\dot{S}$, see Eq. (1.2), to write the expressions in terms of $S_{\alpha S}$.

The relative non-adiabatic energy transfer is given by [11, 12]

$$\delta Q_{\text{rel},\alpha} \equiv -\frac{Q_\alpha}{6H\rho} \sum_\beta \dot{\rho}_\beta S_{\alpha\beta}. \quad (3.15)$$

The non-zero components are

$$\delta Q_{\text{rel},\phi} = \frac{\Gamma \rho_\phi}{6H\rho} (\dot{\rho}_S S_{\phi S} + \dot{\rho}_\gamma S_{\phi\gamma}), \quad (3.16)$$

$$\delta Q_{\text{rel},\gamma} = -\frac{\Gamma \rho_\phi}{6H\rho} (\dot{\rho}_\phi S_{\gamma\phi} + \dot{\rho}_S S_{\gamma S}). \quad (3.17)$$

The evolution of the individual gauge invariant curvature perturbations (on uniform density hypersurfaces) is fed by the non-adiabatic perturbations through [11, 12]

$$\dot{\zeta}_\alpha = \frac{3H^2 \delta P_{\text{intr},\alpha}}{\dot{\rho}_\alpha} - \frac{H \delta Q_{\text{nad},\alpha}}{\dot{\rho}_\alpha}. \quad (3.18)$$

The evolution equations of the curvature perturbations then are

$$\dot{\zeta}_\phi = \frac{\rho_\phi}{3\dot{\rho}_\phi} \left(\dot{\Gamma} - \frac{\Gamma \dot{\rho}_S}{2\rho} \right) S_{\phi S} - \frac{\Gamma \rho_\phi \dot{\rho}_\gamma}{6\rho \dot{\rho}_\phi} S_{\phi\gamma}, \quad (3.19)$$

$$\dot{\zeta}_\gamma = \frac{\rho_\phi}{3\dot{\rho}_\gamma} \left(-\dot{\Gamma} + \frac{\Gamma \dot{\rho}_S}{2\rho} \right) S_{\gamma S} + \frac{\Gamma \dot{\rho}_\phi}{3\dot{\rho}_\gamma} \left(1 - \frac{\rho_\phi}{2\rho} \right) S_{\phi\gamma}, \quad (3.20)$$

$$\dot{\zeta}_S = \frac{3H^2 \delta P_{\text{intr},S}}{\dot{\rho}_S}. \quad (3.21)$$

The equation for $\dot{\zeta}_S$ needs some explanation. First of all, note that the light field S does not undergo energy transfer, unlike the inflaton and the radiation bath. If the equation of state parameter is a constant, as it is for example during S oscillations, the non-adiabatic pressure perturbation vanishes and ζ_S remains constant. This just reflects the fact that $\delta S/S \propto \delta \rho_S / \rho_S$ is constant for a quadratic potential, as S and δS follow the same equation of motion.

During slow roll the equation of state parameter ω is changing with time and the non-adiabatic pressure perturbation is non-zero. If the flat direction field is rolling in a potential

dominated by non-renormalizable terms, this can lead to appreciable damping of $(\delta S/S)$, as discussed in Ref. [5]. Therefore, we will assume a quadratic potential for S with non-renormalizable terms negligible small. This assumption is justified if the VEV of S is small enough, $\frac{1}{2}m_S^2S^2 \ll \lambda^2 S^{2n-2}/M^{2n-6}$, with $n \geq 4$. Small VeVs are naturally obtained in low scale inflation.

However, this still prohibits us from taking the ‘frozen’ limit $\dot{S} \rightarrow 0$ in a naive way. The problem is that in this limit $\omega \rightarrow -1$, and $\zeta_S \rightarrow \infty$ becomes ill defined. To do things properly one has to take the slow roll into account. We write $\omega = -1 + \epsilon$, with $\epsilon = 2T/V$ in the slow roll limit ($\ddot{S} \rightarrow 0$, and $T \ll V$). Here $T = \frac{1}{2}\dot{S}^2$ the kinetic energy, and $V = \frac{1}{2}m_S^2S^2$ is the potential energy. Then

$$P_{\text{intr},S} = -\frac{\dot{\omega}\rho_S}{\dot{\rho}_S}\delta\rho_S = \epsilon\rho_S\frac{\delta\rho_S}{\rho_S}. \quad (3.22)$$

During slow roll the curvature perturbation of S is slowly changing with time.

Eliminating ζ_γ and ρ_γ we can rewrite Eqs. (3.19,3.20) in dimensionless form

$$\begin{aligned} \zeta' &= -\frac{1}{-4 + \Omega_\phi + \Omega_S(1 - 3\omega)} \left\{ \left(-\Omega_\phi(3 + g/h) + 3\Omega_S(1 + \omega)(3\omega - 1) \right) \zeta \right. \\ &\quad \left. - 3\Omega_S(1 + \omega)(3\omega - 1)\zeta_S + (3 + g/h)\Omega_\phi\zeta_\phi \right\}, \end{aligned} \quad (3.23)$$

$$\begin{aligned} \zeta_\phi' &= \left(-4 + \Omega_\phi + \Omega_S(1 - 3\omega) \right) \frac{-g}{2(g + 3h)} \zeta + \frac{g'}{g + 3h} \zeta_S \\ &\quad + \frac{1}{g + 3h} \left(-g' + \frac{g}{2}(-4 + \Omega_\phi + \Omega_S(1 - 3\omega)) \right) \zeta_\phi, \end{aligned} \quad (3.24)$$

where as before a prime denotes differentiation w.r.t. the number of e-foldings $N = \ln a$.

IV. PHYSICAL SITUATIONS

To simplify and explore the properties of the evolution equations for the perturbations we will discuss two special cases in this section.

A. S non-dynamical

First we consider the case where the field S is strictly frozen and is non-dynamical. In addition we require S to have negligible energy density. The evolution of the perturbation is governed by the fluctuating inflaton decay rate, irrespective of what is the cause for

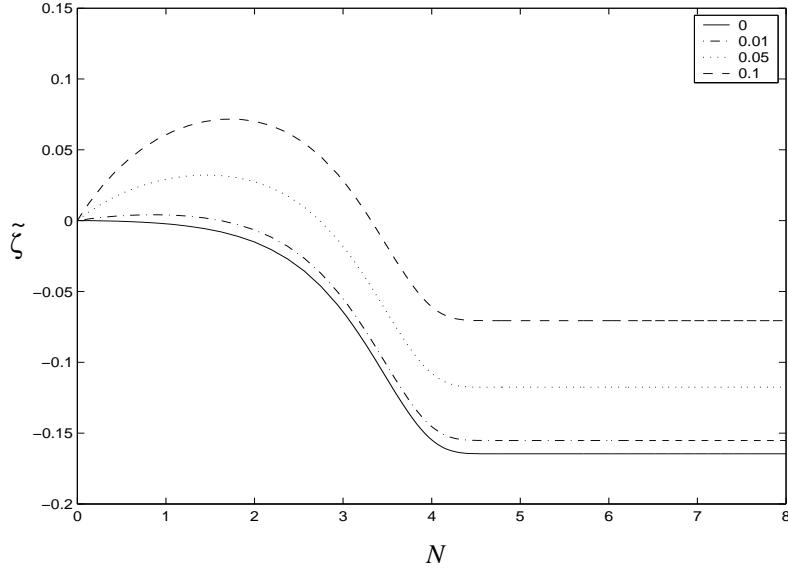


FIG. 1: $\tilde{\zeta}$ as a function of $N = \ln a$ for different initial inflaton perturbations. The bottom line corresponds to $\tilde{\zeta}_\phi(0) = 0$ (solid). The lines above are for $\tilde{\zeta}_\phi(0) = 0.001$ (dashed-dotted), $\tilde{\zeta}_\phi(0) = 0.05$ (dotted), and $\tilde{\zeta}_\phi(0) = 0.1$ (dashed) respectively.

the fluctuations. Note that although ζ_S becomes ill defined in this limit, the combination $g'\zeta_S = \delta\Gamma$ remains finite. The decay rate is independent of time. It is then useful to introduce the quantities

$$f = \frac{\Gamma}{H} = \frac{g}{h}, \quad \delta\Gamma \equiv \frac{\delta\Gamma}{\Gamma}. \quad (4.1)$$

In the non-dynamical limit the background equations become:

$$\Omega'_\phi = \Omega_\phi(1 - f - \Omega_\phi), \quad (4.2)$$

$$f' = -\frac{f}{2}(-4 + \Omega_\phi). \quad (4.3)$$

The perturbation equations are

$$\zeta' = -\frac{(3 + f)\Omega_\phi}{-4 + \Omega_\phi}(\zeta_\phi - \zeta), \quad (4.4)$$

$$\zeta'_\phi = \frac{(-4 + \Omega_\phi)f}{2(3 + f)}(\zeta_\phi - \zeta) - \frac{f\delta\Gamma}{3 + f}. \quad (4.5)$$

We solved the equations numerically. The results are shown in Fig. 1. Plotted is the ratio $\tilde{\zeta} \equiv \zeta/\delta\Gamma$. This quantity is independent of the value of $\delta\Gamma$, as expected [3]. The solid line corresponds to negligible initial curvature perturbation of the inflaton: $\tilde{\zeta}_\phi(N_0 = 0) = 0$,

where we have introduced the notation $\tilde{\zeta}_\alpha \equiv \zeta_\alpha/\delta_\Gamma$. In all plots, $f(0) = 10^{-2}$, i.e., the evolution starts at $H = 10^2 \Gamma$. $H \sim \Gamma$ occurs at $N \approx 3$. We find $\tilde{\zeta} \approx -0.166 = -(1/6)$. This agrees with the results of [3], who found for the gravitational potential during radiation domination $\psi = -(2/3)\zeta = (1/9)\delta_\Gamma$ ².

Furthermore, we looked at the effects of a non-zero initial inflaton curvature. For $\tilde{\zeta}_\phi(0) \lesssim 10^{-3}$ the final curvature is indistinguishable from $\tilde{\zeta}_\phi(0) = 0$. But for larger initial curvatures deviation from $\zeta = -\delta_\Gamma/6$ arise. For $\tilde{\zeta}_\phi(0) = 10^{-2}$ the deviation is $\sim 5\%$, whereas for $\tilde{\zeta}_\phi(0) = 5 \times 10^{-2}$ it is $\sim 30\%$.

B. S oscillating

It is possible that the inflaton decays while the flat direction field is oscillating. This is the case for $\Gamma \lesssim m_S$. Averaged over one oscillation $\omega = 0$, and the averaged decay rate is constant in time. The dimensionless background equations for this case are

$$\Omega'_\phi = \Omega_\phi \left(1 - \frac{g}{h} - \Omega_\phi - \Omega_S \right), \quad (4.6)$$

$$\Omega'_S = \Omega_S (1 - \Omega_\phi - \Omega_S), \quad (4.7)$$

$$f' = -\frac{f}{2}(-4 + \Omega_\phi + \Omega_S). \quad (4.8)$$

The perturbation equations are

$$\zeta' = -\frac{(-\Omega_\phi(3+f) - 3\Omega_S)\zeta + 3\Omega_S\zeta_S + (3+f)\Omega_\phi\zeta_\phi}{-4 + \Omega_\phi + \Omega_S}, \quad (4.9)$$

$$\zeta'_\phi = \frac{(-4 + \Omega_\phi + \Omega_S)f}{2(f+3)}(\zeta_\phi - \zeta) - \frac{f\delta_\Gamma}{3+f}, \quad (4.10)$$

$$\zeta'_S = 0. \quad (4.11)$$

Since S has a fixed equation of state in this case, ζ_S is constant, and it is straightforward to incorporate the evolution of S . It is easy to see that when Ω_S and $\Omega_S \zeta_S$ are negligible, the above equations are the same as in the non-dynamical limit, and the result $\zeta = -\delta_\Gamma/6$ is obtained. This is shown by the solid line in Fig. 2. For Ω_S large enough, the final curvature starts to deviate from this value. A large Ω_S is possible especially in the scenario discussed in [6], where it is not the inflaton but its decay products that have a varying decay rate.

² The metric potentials ψ and ζ are related to each other by $\zeta = -(2/3)(H^{-1}\dot{\psi} + \psi)/(1 + \omega) - \psi$. For constant ψ and $\omega = 1/3$, corresponding to radiation domination, $\zeta = -(3/2)\psi$. For details see Ref. [6, 9].

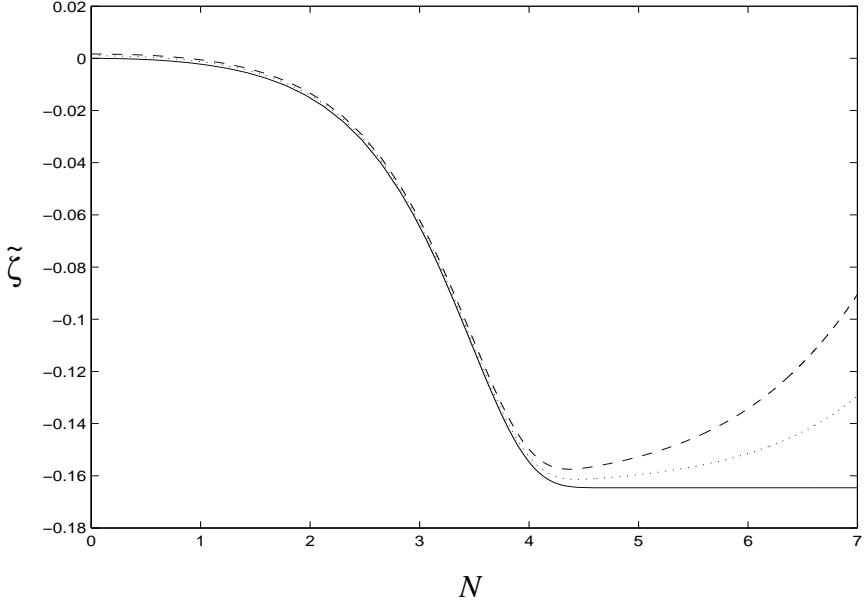


FIG. 2: $\tilde{\zeta}$ as a function of $N = \ln a$ for different contributions of Ω_S and ζ_S . The solid line corresponds to Ω_S negligible small, the dotted line corresponds to $\Omega_S(N = 0) = 10^{-5}$ and $\tilde{\zeta}_S = 10^2$ (for direct decay), and the dashed line corresponds to $\Omega_S(N = 0) = 5 \times 10^{-3}$ and $\tilde{\zeta}_S = 1/3$ (indirect decay). In all plots $\delta_\Gamma = 10^{-5}$.

The field S does not undergo any transfer of energy: $Q_S = 0$. As a result, a non-negligible energy density Ω_S can only affect the total curvature perturbation as a result of non-adiabatic, relative pressure perturbations, see Eq. (3.7). Since we assume that the inflaton red shifts as non-relativistic matter after inflation, there is no relative pressure perturbation between the inflaton and the S field: $S_{\phi S} = 0$. There is a relative pressure perturbation though between the radiation bath and S , and $S_{\gamma S} \neq 0$. However, it is only after inflaton decay that the energy density stored in the radiation bath becomes appreciable, and that Ω_S increases. Then, the relative pressure perturbation leads to the conversion of isocurvature perturbations to curvature perturbations. This is the idea behind the curvaton scenario [2]. The final curvature is approximately given by the function of both ζ_γ and ζ_S :

$$\zeta \approx (1 - r)\zeta_\gamma + r\zeta_S \quad (4.12)$$

with

$$r = \frac{3\Omega_S}{4\Omega_\gamma + 3\Omega_S} \Big|_{S \text{ decay}} \quad (4.13)$$

Here r is to be evaluated at the time of S -decay. Since S does not undergo energy exchange

it acts like a spectator field during inflaton decay, and $\zeta_\gamma \approx -\delta_\Gamma/6$, independently of the value of Ω_S and decay channels, i.e. direct/indirect decay modes, see Eq. (1.2).

The curvature perturbation of the S -field, ζ_S , can be much larger for direct decay than for either indirect decay or the fluctuating mass case, see Eq. (1.3). The curvature perturbation of the flat direction field during the epoch of S -oscillations is

$$\zeta_S = \frac{\delta\rho_S}{3\rho_S} = \frac{2\delta S}{3S}. \quad (4.14)$$

For direct decay $\delta_\Gamma = \delta\Gamma/\Gamma = 2\delta S/M < \zeta_S$, where the last inequality follows from the fact that the cutoff is larger than the VEV of S : $S < M$. A Gaussian perturbation spectrum, as indicated by observations, requires $\delta S \ll S$, and therefore $\zeta_S \ll 1$. For indirect decay $\delta_\Gamma = 3\zeta_S$, whereas for the fluctuating mass case $\delta_\Gamma = 3\zeta_S/2$.

The results for a non-negligible Ω_S are shown in Fig. 2. The solid line corresponds to Ω_S negligible small, when $r\zeta_S \ll (1-r)\zeta_\gamma$, we obtain $\zeta = -\delta_\Gamma/6$ as before. The dotted line is for $\tilde{\zeta}_S = 10^2$ and $\Omega_S \approx 10^{-5}, 2 \times 10^{-5}, 5 \times 10^{-4}$ at $N = 0, 4, 7$ respectively. Such large curvature perturbations of S are possible for direct decay. The dashed line shows the plot for indirect decay, with $\tilde{\zeta}_S = 1/3$ and $\Omega_S \sim 5 \times 10^{-3}, 0.1, 10^{-2}$ at $N = 0, 4, 7$. In both cases the final curvature is well approximated by Eq. (4.12), with $\zeta_\gamma = -\delta_\Gamma/6$. The contribution of ζ_S to the total curvature rises above the one percent level for $(\Omega_S\zeta_S)/\zeta_\gamma \gtrsim 10^{-2}$. For indirect decay and the fluctuating mass case, this is only possible for $\Omega_S \gtrsim 10^{-2}$.

V. CONCLUSIONS

In this paper we presented a gauge invariant formalism to study the evolution of curvature perturbations during decay of the inflaton or some other field that dominates the energy density. We specialized to the case where the perturbations arise from a spatially fluctuating decay rate.

The system we considered consists of the inflaton ϕ , the radiation bath, and a light fluctuating field S which is responsible for the fluctuations in the decay rate of ϕ . During inflaton decay there is an energy transfer from the inflaton field to the radiation bath. Due to the fluctuating decay rate this energy transfer has an intrinsic non-adiabatic component, which feeds the curvature perturbation of the radiation bath. This can be understood intuitively from the fact that fluctuations in the inflaton decay rate leads to fluctuations in

the reheat temperature of the Universe, and therefore to energy fluctuations in the thermal bath $\zeta_\gamma \propto \delta\rho_\gamma/\rho_\gamma \propto \delta\Gamma/\Gamma$. Since after inflaton decay the universe is radiation dominated, the total curvature perturbation is $\zeta \approx \zeta_\gamma$.

We studied the evolution of the background fields and the perturbations numerically. We found that the total curvature perturbation is given by $\zeta = -(1/6)\delta\Gamma$ and $\psi = (1/9)\delta\Gamma$. Here we have assumed that initially ζ_ϕ and Ω_S are negligible small. Departure from $\zeta = -(1/6)\delta\Gamma$, above the one percent level, arises for larger values of $\zeta_\phi(0)/\zeta_\gamma \gtrsim 10^{-2}$ and/or $\Omega_S\zeta_S/\zeta_\gamma \gtrsim 10^{-2}$. It is interesting to note that for indirect decay ζ_S can be large: $\zeta_S \gg \delta\Gamma$ if the VEV of the flat direction field is much less than the cutoff scale $S \ll M$.

VI. NOTE

While we were finishing our work we noticed a related contribution by Matarrese and Riotto, see [13]. In this paper, the gauge invariant curvature perturbations for ϕ, γ are presented. The authors have analytically obtained $\psi = (4/45)\delta\Gamma/\Gamma$, which in our case is proven numerically to be $\psi = (1/9)\delta\Gamma/\Gamma$.

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